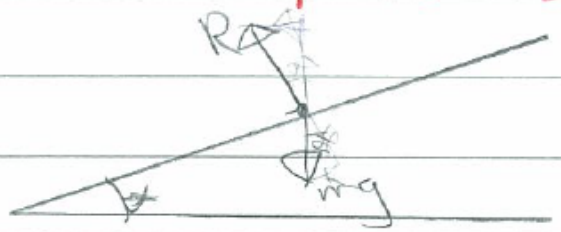
M3 - Specimen Paper

1-



$\leftarrow [F = ma]$

$(R \sin\alpha = m \times 1.4^2 / 100$

$\sin\alpha = 1.96m / R$ (1)

$\uparrow R = mg\cos\alpha$ (2)

(2) in (1):

$\sin\alpha = 1.96m / mg\cos\alpha$

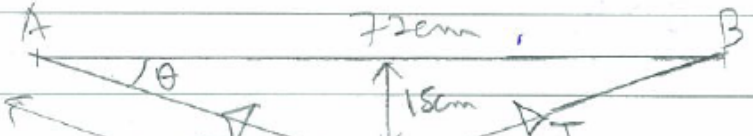
$1.96 = g\sin\alpha\cos\alpha$

$0.4 = \sin 2\alpha$

$2\alpha = 23.6$

$\alpha = 11.8°$ (1dp)

2- a)



$(30 + x)^2 = 15^2 + 36^2$

$$(30+x)^2 = 225+1296$$

$$(30+x)^2 = 1521$$

$$30+x = 39$$

$$x = 9\text{ cm}$$

$$T = \frac{\lambda x}{l} = \frac{9\lambda}{30} = 0.3\lambda$$

$$2g = 2\alpha T \times \frac{15}{39}$$

$$T = \frac{39g}{15}$$

$$T = 0.3\lambda$$

$$\frac{39g}{15} = 0.3\lambda$$

$$\lambda = 84.9\text{N (1dp)}$$

b) No turning effect

3. $\rightarrow [F = ma]$

$$-2x^{-2} = mv\frac{dv}{dx}$$

$$\int_1^x -2x^{-2}\,dx = \int_3^5 0.5v\,dv$$

$$-4\int_1^x x^{-2}\,dx = \int_3^{1.5} v\,dv$$

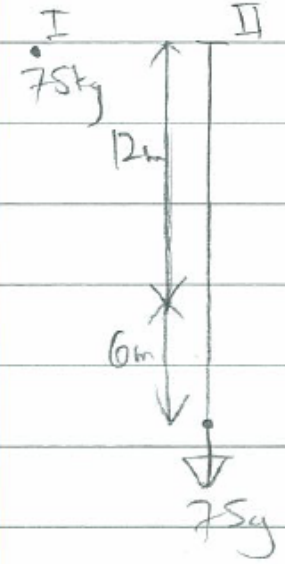
$$-4\left[x^{-1}\right]_1^x = \left[\frac{1}{2}v^2\right]_3^{1.5}$$

$$4x^{-1} - 4 = 1.125 - 4.5$$

$$\frac{4}{x} = 0.625$$

$$x = 6.4\text{ m}$$

4-



a) $mv_I = mv_{II} + WD$

$$75g \times 18 = \frac{\lambda \times 6^2}{24}$$

$$\lambda = \frac{24 \times 75g \times 18^3}{6.6} = 8820\text{N}$$

b) $\frac{8820+36}{24} = \frac{75g}{} + \frac{1}{2} \times 75 \times v^2 + \frac{8820 \times 25}{24}$

$v^2 = 88.2$

$v = 9.39 \text{ ms}^{-1}$ (3sf)

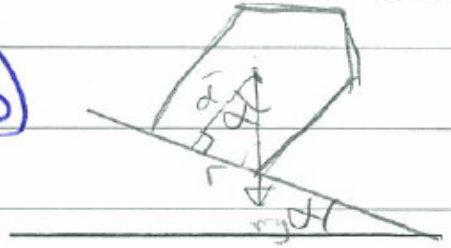
5 a) $\frac{5}{2} \cdot \pi r^2 \cdot r + \left(r + \frac{1}{4}h\right) \cdot \frac{1}{3}\pi r^2 h = \left(\pi r^2 \cdot r + \frac{1}{3}\pi r^2 h\right)\bar{d}$

$3r^2 + 2hr + \frac{1}{2}h^2 = (6r + 2h)\bar{d}$

$6r^2 + 4rh + h^2 = (12r + 4h)\bar{d}$

$\bar{d} = \frac{6r^2 + 4rh + h^2}{4(3r + h)}$

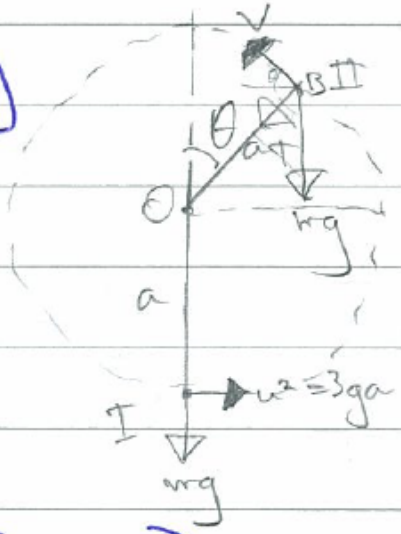
b)



$\tan\alpha = \frac{6r^2 + 4rh + h^2}{4(3r + h) \cdot r} = \frac{6r^2 + 8r^2 + 4r^2}{12r^2 + 8r^2} = \frac{9}{10}$

$\alpha = 48°$

6-a)



$mE_I = mE_{II}$

$\frac{1}{2}m \cdot 3ga = \frac{1}{2}mv^2 + mg(a + a\cos\theta)$

$3ga = v^2 + 2ga + 2ga\cos\theta$

$v^2 = ga - 2ga\cos\theta$

$[F = ma]$

$T + mg\cos\theta = \frac{mv^2}{a}$

$mg\cos\theta = \frac{m}{a}(ga - 2ga\cos\theta)$

$g\cos\theta = g - 2g\cos\theta$

$\cos\theta = \frac{g}{3g} = \frac{1}{3}$

b) $\frac{1}{2}mv^2 = mgh + \frac{1}{2}m(v\cos\theta)^2$, h = greatest height above B

$$\frac{1}{2}ga - ga\cos\theta = gh + \frac{1}{2}\left(\frac{1}{9}ga - \frac{2}{9}ga\cdot\frac{1}{3}\right)$$

$$\frac{1}{2}a - \frac{1}{3}a = h + \frac{1}{2}\cdot\frac{1}{27}a^2$$

$$h = \frac{4a}{27}$$

7- a) $T = mg$

$$T = \frac{\lambda x}{\ell}$$

$$mg = \frac{6mge}{a}$$

$$e = \frac{a}{6}$$

b) $[F = ma]$

$$mg - T = m\frac{d^2x}{dt^2}$$

$$mg - \frac{6mg\left(\frac{a}{6}+x\right)}{a} = m\frac{d^2x}{dt^2}$$

$$g - g - \frac{6gx}{a} = \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{6g}{a}x$$

$$T = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{\frac{6g}{a}}} = 2\pi\sqrt{\frac{a}{6g}}$$

c) $v_{max} = wa = \sqrt{\frac{6g}{a}} \times \frac{1}{3}a = \frac{1}{3}\sqrt{6g} \cdot \frac{a}{\sqrt{a}} = \frac{1}{3}\sqrt{6ga}$

d) $x = a\cos wt$

$$x = \frac{1}{3}a\cos\left(\sqrt{\frac{6g}{a}}\,t\right)$$

$$-\frac{1}{6}a = \frac{1}{3}a\cos\sqrt{\frac{6g}{a}}\,t$$

$$-\frac{1}{2} = \cos\sqrt{\frac{6g}{a}}\,t$$

$$\sqrt{\frac{6g}{a}}\,t = \frac{2\pi}{3}$$

$$t = \frac{2\pi}{3}\sqrt{\frac{a}{6g}}\ s$$